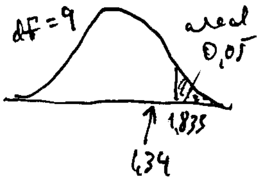
STO201 2008V. Løsningsskisse

1a. $H_0: \mu \le 30$, $H_1: \mu > 30$. $T = \dfrac{\bar{Y} - 30}{S/\sqrt{10}}$ er t-fordelt med 9 frihetsgrader hvis $H_0$ er riktig. Forkaster $H_0$ hvis $T$ er stor.

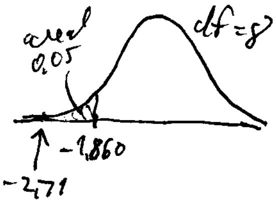



Her: $t = \dfrac{32,16-30}{\sqrt{\frac{1}{9}\cdot 233,324}/\sqrt{10}} = 1,34$. Kritisk verdi: 1,833 (tabell D.5). Forkaster ikke $H_0$.

b. Grenser: $\bar{y} \pm t_{0,025}\dfrac{s}{\sqrt{10}} = 32,16 \pm 2,262\sqrt{\frac{1}{9}\cdot 233,324}/\sqrt{10} = 32,16 \pm 3,64$ (df=9).
Konf.int. $[28,5,\ 35,8]$

c. $\hat{\beta} = \dfrac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2} = \dfrac{-105,88}{100,465} = -1,05$, $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 32,16-(-1,0539)\cdot 64,05 = 99,66$

$H_0: \beta \ge 0$, $H_1: \beta < 0$. $T = \dfrac{\hat{\beta}}{\sqrt{\frac{SSE}{10-2}/\sum(x_i-\bar{x})^2}}$ er t-fordelt med 8 frihetsgrader hvis $H_0$ er riktig. Forkaster $H_0$ hvis $T$ er liten.




Her: $t = \dfrac{-1,0539}{\sqrt{\frac{121,737}{8}/100,465}} = -2,71$. Kritisk verdi: $-1,860$.

Forkaster $H_0$, grunn til å påstå at halelengden minker nordover.

d. Bruker Fishers transformasjon. $r = \dfrac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2}\sqrt{\sum(y_i-\bar{y})^2}} = \dfrac{-105,88}{\sqrt{100,465}\sqrt{233,324}} = -0,692$,

$z = \frac{1}{2}\ln\dfrac{1+r}{1-r} = \frac{1}{2}\ln\dfrac{1-0,692}{1+0,692} = -0,851$. 95%-konfidensintervall:

$$\left[\frac{e^{2(z-u_{\alpha/2}/\sqrt{n-3})}-1}{e^{2(z-u_{\alpha/2}/\sqrt{n-3})}+1},\ \frac{e^{2(z+u_{\alpha/2}/\sqrt{n-3})}-1}{e^{2(z+u_{\alpha/2}/\sqrt{n-3})}+1}\right].$$

Med $u_{\alpha/2} = 1,960$ og $n=10$ blir det

$[-0,92,\ -0,11]$.

2a. $E\hat{\mu} = \frac{2}{3}(E\bar{X} + E\bar{Y}) = \frac{2}{3}(\mu + \frac{1}{2}\mu) = \mu$, $E\mu^* = \frac{1}{2}E\bar{X} + E\bar{Y} = \frac{1}{2}\mu + \frac{1}{2}\mu = \mu$.

$\mathrm{Var}\,\hat{\mu} = \frac{4}{9}(\mathrm{Var}\,\bar{X} + \mathrm{Var}\,\bar{Y}) = \frac{4}{9}\left(\frac{\mu^2}{8} + \frac{(\mu/2)^2}{4}\right) = \frac{1}{12}\mu^2$

$\mathrm{Var}\,\mu^* = \frac{1}{4}\mathrm{Var}\,\bar{X} + \mathrm{Var}\,\bar{Y} = \frac{1}{4}\frac{\mu^2}{8} + \frac{(\mu/2)^2}{4} = \frac{3}{32}\mu^2$

Begge er forventningsrette. Foretrekker $\hat{\mu}$, som har minst varians
$\left(\frac{3}{32} > \frac{3}{36} = \frac{1}{12}\right)$.

b. Likelihoodfunksjon: $L = \prod_{i=1}^{8}\left(\frac{1}{\mu}e^{-x_i/\mu}\right)\cdot\prod_{j=1}^{4}\left(\frac{2}{\mu}e^{-2y_j/\mu}\right) = 16\mu^{-12}e^{-(\sum x_i + 2\sum y_j)/\mu}$

$\ln L = \text{konst.} - 12\ln\mu - \dfrac{\sum x_i + 2\sum y_j}{\mu}$, $\dfrac{d\ln L}{d\mu} = -\dfrac{12}{\mu} + \dfrac{\sum x_i + 2\sum y_j}{\mu^2}$,

$\dfrac{d\ln\mu}{d\mu} = 0$: $\hat{\mu} = \dfrac{\sum x_i + 2\sum y_j}{12} = \dfrac{2}{3}\dfrac{\sum x_i}{8} + \dfrac{2}{3}\dfrac{\sum y_j}{4} = \dfrac{2}{3}(\bar{x} + \bar{y})$.

Sannsynlighetsmaksimeringsestimator: $\hat{\mu} = \frac{2}{3}(\bar{X} + \bar{Y})$.