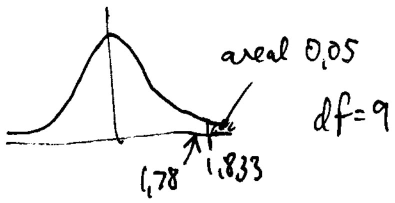
ST0201 2007H, løsningsskisse

1a. $\hat{\mu} = \bar{x} = \frac{23,29}{10} = 2,329$. 95%-konfidensintervall har grenser $\bar{x} \pm z_{\alpha/2}\,\sigma/\sqrt{n}$
$= 2,329 \pm 1,960 \cdot 0,5/\sqrt{10} = 2,329 \pm 0,310$. Konfidensintervallet: [2,02, 2,64].

b. $1,960 \cdot 0,5/\sqrt{n} < 0,1$, dvs. $n > \left(\frac{1,960 \cdot 0,5}{0,1}\right)^2 = 96,04$, $n \geq 97$.

c. $H_0: \mu \leq 2$. $H_1: \mu > 2$. $z = \frac{2,329-2}{0,5/\sqrt{10}} = 2,08$, $P(Z > 2,08) = 0,0188$. Forkast H_0.

σ ukjent: $s = \sqrt{\frac{3,085}{9}} = 0,585$, $t = \frac{2,329-2}{0,585/\sqrt{10}} = 1,78$. Kritisk verdi 1,833,



forkaster ikke H_0.

d. Vi forkaster H_0 hvis $\frac{\bar{X}-2}{0,5/\sqrt{10}} > 1,645$. Hvis $\mu = 2,1$:

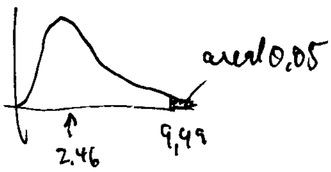
$$P\left(\frac{\bar{X}-2}{0,5/\sqrt{10}} > 1,645\right) = P\left(\frac{\bar{X}-2,1}{0,5/\sqrt{10}} > 1,645 - \frac{0,1}{0,5/\sqrt{10}}\right) = P(Z > 1,01) = 0,1562.$$

2a.

Observert antall	55	19	12	8	6
Forventet antall	50	25	12,5	6,25	6,25

(slår sammen cellene som svarer til ≥ 5 unger for å få forv. antall ≥ 5.)

$$\chi^2 = \frac{(55-50)^2}{50} + \frac{(19-25)^2}{25} + \frac{(12-12,5)^2}{12,5} + \frac{(8-6,25)^2}{6,25} + \frac{(6-6,25)^2}{6,25} = 2,46$$



$2,46 < 9,49$, forkaster ikke nullhypotesen om at antall unger er geometrisk fordelt med parameter 0,5.

b. Likelihoodfunksjon: $L = \prod_{i=1}^{100} (1-p)^{x_i-1} p$, $\ln L = \sum_{i=1}^{100} (x_i - 1)\ln(1-p) + 100 \ln p$

$\frac{d \ln L}{dp} = -\frac{\sum x_i - 100}{1-p} + \frac{100}{p}$, $\frac{d \ln L}{dp} = 0$: $\frac{\sum x_i - 100}{1-p} = \frac{100}{p}$, som gir $p = \frac{100}{\sum x_i}$.

Sannsynlighetsmaksimeringsestimat: $\hat{p} = \frac{100}{55\cdot1 + 19\cdot2 + 12\cdot3 + 8\cdot4 + 3\cdot5 + 3\cdot6} = 0,515$.