English

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ST0101 Probability with applications

Thursday 29 November 2007
9:00–13:00
Permitted aids: Any written and printed material. One calculator.
Grades to be announced: 20 December 2007

The final examination consists of two parts:

1. The problems on the next page.

2. Appendix with a multiple choice questionnaire.

The Appendix is to be submitted with the form filled in together with the answer to part (1). Part (1) and (2) count equally in the evaluation of the final examination.

In addition to the final examination the mid-term examination counts 20% if it is advantageous to the candidate.

In the evaluation of part (1) (next page) each of the eight points counts equally.

In part (1) you should demonstrate how you arrive at your answers (e.g. by including intermediate answers or referral to theory). Answers based on calculator or table look-up only will not be accepted.

Problem 1

A chemist produces a substance by a procedure that gives a yield of X mg of the substance, where X is normally distributed with mean $\mu = 12$ and standard deviation $\sigma = 2$. An instrument is used in the production. A warning lamp lights if the yield is less than 10.5 mg.

a) Find the probability that the warning lamp lights.

b) The procedure is performed twice, and the yields are assumed to be independent of each other. Find the probability that the smallest of the two yields is less than 10.5 mg.

c) The instrument can be adjusted so that the procedure is repeated as many times as requested, n. The yields are independent of each other. The instrument is adjusted so that the procedure is repeated $n = 100$ times. Find an approximate probability that the lamp lights 25 or more times.

d) (*Difficult.*) How large must n at least be for the total yield to be at least 100 mg with probability 0.999 or more?

Problem 2

A couple that wants to establish a family, decides to have children until they have a girl. Let X be the number of children they get. We assume that the can have an unlimited number of children, if necessary. Assume that the probability of having a boy is 1/2 and that the probability of having a girl is 1/2.

a) Argue that X is geometrically distributed.

b) What is the expected value of X? What is the expected value of the number of boys and of the number of girls (*hint:* first determine the expected value of the number of girls)?

c) Find the conditional probabilities $P(X = 1 \mid X \leq 2)$ and $P(X = 2 \mid X \leq 2)$. Find the conditional expected value of the number of children given that the number is two or less, that is $E(X \mid X \leq 2)$. What is the conditional expected value of the number of boys and of the number of girls?

d) Another family also decides to have children until they have a girl, but stop if they have three children, irrespective of the fact that all are boys. What is the expected number of children, the expected number of boys and the expected number of girls in this case?