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Department of Mathematical Sciences

English

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ST0101 Probability with applications

Monday 11 December 2006
9:00–13:00
Permitted aids: Any written and printed material. One calculator.
Grades to be announced: 11 January 2007

The final examination consists of two parts:

1. The problems on the next page.
2. Appendix with a multiple choice questionnaire.

The Appendix is to be submitted with the form filled in together with the answer to part (1). Part (1) and (2) count equally in the evaluation of the final examination.

In addition to the final examination the mid-term examination counts 20% if it is advantageous to the candidate.

In the evaluation of part (1) (next page) each of the eight points counts equally.

In part (1) you should demonstrate how you arrive at your answers (e.g. by including intermediate answers or referral to theory). Answers based on calculator or table look-up only will not be accepted.

Problem 1

A randomly chosen nest of a bird species contains X eggs. The probability distribution of X is as follows:

x	1	2	3	4	5	6
$P(X = x)$	0.09	0.38	0.25	0.20	0.06	0.02

Assume that the number of eggs in each nest is independent of each other.

a) Find the expected value and standard deviation of X.

b) Find the probability that a randomly chosen nest has 4 or more eggs. Find the conditional probability that a randomly chosen nest has 6 eggs given that it has 4 or more eggs.

c) We examine 50 randomly chosen nests. Let Y be the number of these having 4 or more eggs. What is the expected value and standard deviation of Y? Find an approximate probability that 10 or more of the 50 nests have at least 4 eggs.

d) If a nest of this bird species is built in a deciduous tree, the probability is 0.05 that it contains only one egg. If a nest is built in a coniferous tree, the probability is 0.15 that it contains only one egg. The bird species always build the nest in a deciduous tree or in a coniferous tree. What is the probability that a randomly chosen nest is built in a deciduous tree?

Problem 2

At a research station there is an insect cage containing a large number of flies of a certain species. Half are male and half are female. If the door of the cage is kept open, a randomly chosen fly escapes after T minutes, and $P(T > t) = e^{-0.5t}$ for males and $P(T > t) = e^{-0.3t}$ for females, where $t > 0$. We assume that the escape times of the flies are independent of each other.

a) Find the probability that a randomly chosen male uses more than one minute to escape. Find the same probability for a randomly chosen female.

b) Find the probability that a randomly chosen fly uses more than one minute to escape.

c) Find the conditional probability that a randomly chosen fly is a male given that it uses more than one minute to escape.

d) The researchers want to reduce the proportion of males among all the flies to 1/3 by keeping the cage open for a certain amount of time. How long must the cage be open for the expected number of males left in the cage to be half of the expected number of females left? What proportion of the flies is expected to have escaped then?