Mid-term examination in MA0301 Elemetary Discrete Mathematics
Tuesday 9 March 2010 8:15–9:45

Permitted aid: Approved calculator.

Mark one answer for each problem on the form overleaf. You will score one point for each right answer and zero points for each wrong answer. Multiple answers will score zero.

NB: There is text on both sides of the sheet. All problems have five alternative answers.

Problem 1. Define the connective $\uparrow$ by $(p \uparrow q) \Leftrightarrow \neg(p \wedge q)$. Which statement is logically equivalent with $(p \uparrow q) \uparrow (p \uparrow q)$?

(a) $p \vee q$ **(b)** $\neg p$ **(c)** $p \wedge q$ **(d)** $p \leftrightarrow q$ **(e)** $p \rightarrow q$

Problem 2. What is the coefficient in front of ab^5c^5 when we expand $(a - 3b + c)^{11}$?

(a) -112266 **(b)** -213444 **(c)** -6075 **(d)** -462 **(e)** -673596

Problem 3. There are four houses in a line. Each house is to be painted white, red or yellow. In how many ways can this be achieved if neighbouring houses should not have the same colour?

(a) 24 **(b)** 81 **(c)** 16 **(d)** 36 **(e)** 108

Problem 4. How many subsets does $\{\, x \in \mathbb{Z} \mid 0 \leq x \leq 9 \,\}$ have?

(a) 45 **(b)** 1024 **(c)** 512 **(d)** 10 **(e)** 256

Problem 5. Let x and y be real numbers. Which statement is true?

(a) $\forall x \, \exists y \, (x > y^2)$

(b) $\forall x \, \forall y \, (x > y)$

(c) $\forall x \, \exists y \, (x > y)$

(d) $\exists x \, \forall y \, (x > y^2)$

(e) $\exists x \, \forall y \, (x > y)$

Problem 6. The function $f\colon \mathbb{Z}\times(\mathbb{Z}-\{0\}) \rightarrow \mathbb{Q}$ is given by $f(a, b) = a/b$ for all $(a, b) \in \mathbb{Z}\times(\mathbb{Z}-\{0\})$. Which statement is true?

(a) f is neither one-to-one nor onto $\mathbb{Q}$

(b) f is one-to-one, but not onto $\mathbb{Q}$

(c) f is both one-to-one and onto $\mathbb{Q}$

(d) f cannot be defined in this way

(e) f is onto $\mathbb{Q}$, but not one-to-one

Problem 7. We define the sequence a_1, a_2, a_3, ... of numbers recursively by $a_1 = 0$ and $a_n = n + a_{n-1}$ for all integers $n \geq 2$. Find a_5.

(a) 10 **(b)** 15 **(c)** 14 **(d)** 6 **(e)** 9

Problem 8. We consider the same sequence as in the previous problem. Find a_{100}.

(a) 2524 **(b)** 300 **(c)** $2^{99} - 1$ **(d)** 5049 **(e)** 105

Problem 9. A committee of two girls and two boys is to be elected in a class of 11 girls and 12 boys. In how many ways can the committee be elected?

(a) 17424 **(b)** 528 **(c)** 3630 **(d)** 8855 **(e)** 14520

Problem 10. In how many ways can six distinct objects be distributed into three identical containers, when no container is left empty?

(a) 120 **(b)** 56 **(c)** 165 **(d)** 90 **(e)** 729

Problem 11. Which of the following sets is *not* necessarily equal to the others?

(a) $(A - B) \cup (B - A)$

(b) $A \cup (B - A)$

(c) $(A \cap \overline{B}) \cup (A \cap B) \cup (\overline{A} \cap B)$

(d) $A \cup B$

(e) $(A \cap B) \cup (A \triangle B)$

Problem 12. Which statement is a tautology?

(a) $\neg(p \vee \neg q) \to \neg p$

(b) $(p \to q) \to q$

(c) $(p \to q) \to (q \to p)$

(d) $(p \to q) \to p$

(e) $q \leftrightarrow (\neg p \vee \neg q)$

Problem	a	b	c	d	e
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Candidate number

Programme of study

Inspector