English

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MA0301 Elementary discrete mathematics

Monday 6 December 2010 9:00–13:00

Permitted aids: No printed or written aids permitted. Calculator HP 30s or Citizen SR-270X

Grades to be announced: 6 January 2011

In the grading each of the ten problems counts equally.

In addition to the final examination the mid-term examination counts 20% if it is advantageous to the candidate.

Unless otherwise stated, **you should demonstrate how you arrive at your answers** (e.g. by including intermediate answers or by referring to theory or examples from the reading list).

Problem 1

In how many ways is it possible to select a committee consisting of two girls and two boys in a school class of 13 girls and 17 boys?

Problem 2

What is the coefficient of $x^{97}y^3$ in the expansion of $(x - y)^{100}$?

Problem 3

Is $(p \wedge (q \vee (p \to r))) \leftrightarrow ((p \wedge q) \vee r)$ a tautology?

Problem 4

Given a finite set A, where $1 \in A$, and where 1 is an element in a third of all subsets of cardinality 4 of A. What is the cardinality of A?

Problem 5

Show *by induction* the the sum of the n smallest positive odd numbers is n^2.

Problem 6

The function $f \colon \mathbb{Z} \to \mathbb{Z}$ is defined by $f(x) = (3x+1)^2$ for all $x \in \mathbb{Z}$. Is f one-to-one (injective)? Is f onto $\mathbb{Z}$ (surjective)?
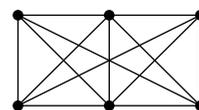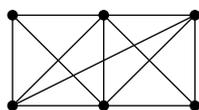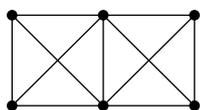
Problem 7

Construct a finite state machine having both input alphabet and output alphabet $\{0, 1\}$, giving output 1 whenever two consecutive input symbols are different and output 0 otherwise. For example, the input string 11101001 should give the output string 00011101.

Problem 8

Given the relation $\mathcal{R} = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid a - b \text{ is divisible by } 3\}$ on $\mathbb{Z}$. Determine whether $\mathcal{R}$ is reflexive, symmetric, antisymmetric and/or transitive. Find the equivalence classes of $\mathcal{R}$ if $\mathcal{R}$ is an equivalence relation.

Problem 9

Determine which of the three graphs (if any) are planar (the vertices are the marked points).

Problem 10

Find the shortest path from a to f and its length by applying Dijkstra's algorithm. You do not have to show how you arrive at the answer, but you should write down all labels at the vertices (from left to right or from top to bottom at each vertex).