



English

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MA0002 Mathematical methods B

Saturday 10 June 2006

9:00–13:00

Permitted aids: Any written and printed material. One calculator.

Grades to be announced: 1 July 2006

The final examination consists of two parts:

1. The problems on the next page.
2. Appendix with a multiple choice questionnaire.

The Appendix is to be submitted with the form filled in together with the answer to part (1). Part (1) and (2) count equally in the evaluation of the final examination.

In addition to the final examination the mid-term examination counts 20% if it is advantageous to the candidate.

In the evaluation of part (1) (next page) each of the eight points counts equally.

In part (1) you should demonstrate how you arrive at your answers (e.g. by including intermediate answers or referral to theory). Answers based on calculator or table look-up only will not be accepted.

Problem 1

A laboratory investigates algal growth at a temperature around the freezing point of water. The algae grow on a circular plate of radius 1 dm. The temperature measured in °C at the point (x, y) on the plate is

$$f(x, y) = \frac{4(x + 1)}{1 + 4(x + 1)^2 + y^2},$$

where x and y are the coordinates measured in decimetres in a coordinate system with origin in the centre of the circular plate, so that $x^2 + y^2 \leq 1$.

- a) Find the partial derivatives of f .
- b) In which direction from the point $(0, 0)$ does the temperature increase most rapidly?
- c) Find critical points (x, y) of f in the interior of the domain (i.e. such that $x^2 + y^2 < 1$). You are not required to investigate which type of critical point it is.
- d) Find the highest and lowest temperature on the plate. (The calculations may be somewhat laborious.)

Problem 2

Given the matrix

$$M = \begin{bmatrix} 0.5 & 2 \\ 0.5 & 0.5 \end{bmatrix}.$$

- a) Find the inverse matrix of M .
- b) Find the eigenvalues of M .
- c) Find an eigenvector for each of the eigenvalues.

In a bird population there are 10 hatchlings and 10 adults. After n years the numbers of hatchlings and adults are the first and the second entry of $M^n \begin{bmatrix} 10 \\ 10 \end{bmatrix}$, respectively.

- d) What is the ratio of the number of hatchlings to the number of adults after several years? What is the population's long-term relative growth rate?