Mid-term examination in MA0002 Mathematical methods B
Wednesday 9 March 2005 14:15–16:00

Permitted aids: Any written and printed material. One calculator.

Mark one answer for each problem on the form overleaf. You will score one point for each right answer and zero points for each wrong answer. Multiple answers will score zero.

NB: There is text on both sides of the sheet. All problems have five alternative answers.

Problem 1. Let y be a solution of the initial value problem $dy/dx + y = e^x$, where $y = 1$ when $x = 0$. What is y equal to when $x = 1$?

(a) $\frac{1}{2}(1 + e^{-1})$ **(b)** $\frac{1}{2}(e - e^{-1})$ **(c)** $\frac{1}{2}(e + 1)$ **(d)** $\frac{1}{2}(1 - e^{-1})$ **(e)** $\frac{1}{2}(e + e^{-1})$

Problem 2. Which types of equilibrium points does the differential equation $dx/dt = (x - 1)(x - 2)(x - 4)$ have?

(a) One stable and two unstable **(b)** One stable and one unstable **(c)** Three stable **(d)** The equation has no equilibrium point **(e)** Two stable and one unstable

Problem 3. In an animal population with x_1 juveniles and x_2 adults the number of juveniles next year is $x_1 + 1.5x_2$ and the number of adults next year is $0.5x_1$. If $\mathbf{x} = \left[\begin{smallmatrix} x_1 \\ x_2 \end{smallmatrix}\right]$ and $L = \left[\begin{smallmatrix} 1.0 & 1.5 \\ 0.5 & 0 \end{smallmatrix}\right]$, then the number of juveniles and the number of adults after n years are given by $L^n\mathbf{x}$. What is the ratio of the number of juveniles to the number of adults in the long run?

(a) 4 : 3 **(b)** 3 : 2 **(c)** 3 : 4 **(d)** 1 : 3 **(e)** 3 : 1

Problem 4. We consider the same animal population as in the last problem. What is the population's relative growth rate in the long run?

(a) 0.6 **(b)** 1.0 **(c)** 0.8 **(d)** 1.5 **(e)** 1.2

Problem 5. Assume that x, y and z satisfy the system $x + y + 2z = 9$, $2x + 4y - 3z = 1$, $3x + 6y - 5z = 0$ of equations. Which of the following is true?

(a) $y = 5$ **(b)** $y = -5$ **(c)** The system of equations has infinitely many solutions **(d)** $y = 2$ **(e)** The system of equations has no solution

Problem 6. In a chemical reaction the concentration x of a substance decreases with a rate proportional to the square root of the concentration, i.e. $dx/dt = -kx^{1/2}$, where $k > 0$ is a constant and t is time. Let the initial concentration (at $t = 0$) be x_0. How long will it take for the concentration to decrease to the half?

(a) $x_0/((\sqrt{2} - 1)k)$ **(b)** x approaches $x_0/\sqrt{2}$ asymptotically **(c)** $\ln(2 + \sqrt{2})/k$ **(d)** $2(\sqrt{2} - 1)/(k\sqrt{x_0})$ **(e)** $(2 - \sqrt{2})\sqrt{x_0}/k$

Problem 7. What is $\left[\begin{smallmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{smallmatrix}\right] \left[\begin{smallmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{smallmatrix}\right]$ equal to?

(a) The matrix multiplication is not defined **(b)** $\left[\begin{smallmatrix} 3 & 10 \\ 13 & 7 \end{smallmatrix}\right]$ **(c)** $\left[\begin{smallmatrix} 3 & 0 \\ 3 & 5 \end{smallmatrix}\right]$ **(d)** $\left[\begin{smallmatrix} 3 & 6 & 0 \\ -2 & 5/2 & 4 \\ 3 & 1 & 5 \end{smallmatrix}\right]$ **(e)** $\left[\begin{smallmatrix} 3 & 12 & 6 \\ 5 & -2 & 8 \\ 4 & 5 & 7 \end{smallmatrix}\right]$

Problem 8. What is *not* an eigenvector of $\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$?

(a) $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}'$ **(b)** $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}'$ **(c)** $\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}'$ **(d)** $\begin{bmatrix} -1 & 1 & 1 \end{bmatrix}'$ **(e)** $\begin{bmatrix} -2 & 1 & 1 \end{bmatrix}'$

Problem 9. What is the inverse matrix of $\begin{bmatrix} 3 & -2 \\ 3 & -1 \end{bmatrix}$?

(a) $\frac{1}{3}\begin{bmatrix} 1 & -2 \\ 3 & -3 \end{bmatrix}$ **(b)** The matrix is not invertible **(c)** $\frac{1}{3}\begin{bmatrix} -3 & 3 \\ -2 & 1 \end{bmatrix}$ **(d)** $\frac{1}{3}\begin{bmatrix} -1 & 2 \\ -3 & 3 \end{bmatrix}$ **(e)** $\frac{1}{3}\begin{bmatrix} 3 & -3 \\ 2 & -1 \end{bmatrix}$

Problem 10. Calculate the determinant $\begin{vmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{vmatrix}$.

(a) 0 **(b)** -65 **(c)** 56 **(d)** -56 **(e)** 65

Problem	a	b	c	d	e
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Studentnummer Student number

Studieprogram Study program

Inspektør Inspector