Norwegian University
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Department of Mathematical Sciences

English

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MA0002 Mathematical methods B

Tuesday 24 May 2005
9:00–13:00
Permitted aids: Any written and printed material. One calculator.
Grades to be announced: 14 June 2005

The final examination consists of two parts:

1. The problems on the next page.
2. Appendix with a multiple choice questionnaire.

The Appendix is to be submitted with the form filled in together with the answer to part (1). Part (1) and (2) count equally in the evaluation of the final examination.

In addition to the final examination the mid-term examination counts 20% if it is advantageous to the candidate.

In the evaluation of part (1) (next page) each of the eight points counts equally.

In part (1) you should demonstrate how you arrive at your answers (e.g. by including intermediate answers or referral to theory). Answers based on calculator or table look-up only will not be accepted.

Problem 1

At a biological research station there is an artificial terrain formation. On a square of 30×30 meters the altitude of the terrain in meters (measured from a reference level) is $f(x, y) = 81/(27 + x^2 + y^2)$, where x and y are the coordinates measured in meters, $-15 \leq x \leq 15$, $-15 \leq y \leq 15$.

a) Sketch the level curves of f corresponding to the altitudes 0.5, 1, 1.5, 2, 2.5 and 3 meters.

b) Find the gradient of f. In which direction from the point $(5, 10)$ is the downhill slope steepest?

c) What is the largest and smallest altitude of the terrain on the square $-15 \leq x \leq 15$, $-15 \leq y \leq 15$?

d) (Difficult.) Consider a point at a distance t meters from the centre of the square. What is the maximal slope at this point? At which distance from the centre of the square is the maximal slope largest, and what is the altitude there?

Problem 2

a) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 6 & -2 \\ 4 & -3 \end{bmatrix}.$$

b) Find the equilibrium point of the system

$$(*) \qquad \begin{aligned} dx/dt &= 6x - 2y - 8 \\ dy/dt &= 4x - 3y + 3 \end{aligned}$$

of differential equations. Is the equilibrium point stable or unstable?

c) Find the general solution of the system

$$\begin{aligned} dX/dt &= 6X - 2Y \\ dY/dt &= 4X - 3Y \end{aligned}$$

of differential equations.

d) The general solution of the system $(*)$ of differential equations in **b** is $x = X + a$ and $y = Y + b$, where (a, b) is the equilibrium point of $(*)$ (you are not required to show this). Find the particular solution of $(*)$ satisfying $x = 4$ and $y = 2$ when $t = 0$.