MA0001, 2003H. Løsningsskisse

1(a) $f(x) = xe^{-x}$, $f'(x) = 1\cdot e^{-x} + x\cdot(-e^{-x}) = -xe^{-x} + e^{-x} = -(x-1)e^{-x}$,

$f''(x) = -\left(e^{-x} + (x-1)(-e^{-x})\right) = (x-2)e^{-x}$, $f'''(x) = e^{-x} + (x-2)(-e^{-x}) = -(x-3)e^{-x}$.

$f(x) < 0$ for $x<0$, $f(x)=0$ for $x=0$, $f(x)>0$ for $x>0$. Nullpunkt: 0

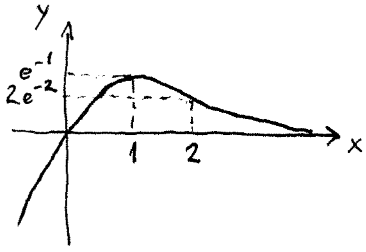
$f'(x) > 0$ for $x<1$, $f'(x)=0$ for $x=1$, $f'(x)<0$ for $x>1$. Abs. maks. i 1

$f''(x) < 0$ for $x<2$, $f''(x)=0$ for $x=2$, $f''(x)>0$ for $x>2$. Vendepunkt i 2.

(b) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} xe^{-x} = \lim\limits_{x\to\infty} \frac{x}{e^x}$

$= \lim\limits_{x\to\infty} \frac{1}{e^x} = 0$ (l'Hôpitals regel)

(c)



(d) arealet $= \int_0^\infty f(x)\,dx = \int_0^\infty xe^{-x}\,dx = \lim\limits_{t\to\infty} \int_0^t xe^{-x}\,dx$ | delvis int., $u=x$, $v'=e^{-x}$, $u'=1$, $v=-e^{-x}$

$= \lim\limits_{t\to\infty}\left([-xe^{-x}]_0^t + \int_0^t e^{-x}\,dx\right) = \lim\limits_{t\to\infty}\left(-te^{-t} + 0 - [e^{-x}]_0^t\right)$

$= \lim\limits_{t\to\infty}\left(-te^{-t} - e^{-t} + 1\right) = 0 + 0 + 1 = 1.$

2(a) $f(0)=0$, $f'(0)=1$, $f''(0)=-2$. Andregrads taylorpolynom:

$P(x) = 0 + 1x - \frac{2}{2}x^2 = x - x^2$. $f(0{,}1) \approx P(0{,}1) = 0{,}1 - 0{,}1^2 = 0{,}09$.

(b) Det fins z mellom 0 og x slik at $f(x) = x - x^2 + \frac{f'''(z)}{3!}x^3 = x - x^2 + \frac{(3-z)e^{-z}}{6}x^3$,

så det fins z mellom 0 og 0,1 slik at $f(0{,}1) = 0{,}09 + 0{,}001\frac{(3-z)e^{-z}}{6}$.

Da er $3-z \le 3$ og $e^{-z} \le 1$, dermed $0 < 0{,}001\frac{(3-z)e^{-z}}{6} \le 0{,}001\cdot\frac{3\cdot 1}{6} = 0{,}0005$.

Altså er $f(0{,}1) = 0{,}09 + d$, der $0 < d \le 0{,}0005$, og feilen vi gjør er høyst 0,0005.

[Med 5 desimaler: $f(0.1) = 0.1\cdot e^{-0{,}1} = 0{,}09048$, dvs. feilen er 0,00048 med 5 desimaler. Dette ut fra kalkulatorverdi.]